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EVEN VERTEX EQUITABLE EVEN LABELING FOR TREE RELATED GRAPHS

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Abstract. Let G be a graph with p vertices and q edges and $A = \{0, 2, 4, \cdots, q+1\}$ if q is odd or $A = \{0, 2, 4, \cdots, q\}$ if q is even. A graph G is said to be an even vertex equitable even labeling if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2, 4, \cdots, 2q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits even vertex equitable even labeling is called an even vertex equitable even graph. In this paper, we prove that $T\hat{o}Q_n$, $T\tilde{o}Q_n$ and H-graph are an even vertex equitable even graphs.

Keywords: Vertex equitable labeling, even vertex equitable even labeling, tree.

1. Introduction

All graphs considered here are simple, finite, connected and undirected. The vertex set and the edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively. We follow the basic notations and terminology of graph theory as in [2]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices then the labeling is called vertex labeling. If the domain is the set of edges then the labeling is called edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. For a dynamic survey of various graph labeling, we refer to Gallian [1].

Lourdusamy *et al.* introduced the concept of vertex equitable labeling in [20]. Let G be a graph with p vertices and q edges and let $A = \{0, 1, 2, \cdots, \lceil \frac{q}{2} \rceil\}$. A vertex labeling $f : V(G) \to A$ induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv.

For $a \in A$, let $v_f(a)$ be the number of vertices v with $f(v) = a$. A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, \cdots, q$.

Motivated by the concept of vertex equitable labeling and further results by Jeyanthi *et al.* in [4, 5, 6, 7, 8, 9, 10, 11]. Lourdusamy *et al.* introduced the concept of even vertex equitable even labeling [18]. In [12, 13, 14, 15, 16, 17, 18, 19], they proved that path, comb, complete bipartite, cycle, $K_2 + mK_1$, bistar, ladder, $S(L_n)$, $S(B_{n,n})$, $L_n \odot K_1$, P_n^2, $S(P_n \odot K_1)$, $S^{'}(P_n)$, $T(P_n)$, graph obtained by duplication of each vertex by an edge in P_n, Q_n, $S(Q_n)$, $D(Q_n)$, $A(T_n)$, $DA(T_n)$, $P_n \odot mK_1$, $P_n(Q_m)$, $S^{*}(P_n \odot K_1)$, $S^{*}(L_n)$, $S^{*}(B_{n,n})$, $B^2_{n,n}$, $S^{'}(B_{n,n})$, $L_n \odot mK_1$, $C_n \odot K_1$, T_p-tree, $T\hat{o}P_n$, $T\hat{o}2P_n$, $T\hat{o}C_n (n \equiv 0, 3 (mod\, 4))$, $T\tilde{o}C_n (n \equiv 0, 3 (mod\, 4))$, $T\hat{o}K_{1,n}$, $T \odot \overline{K_n}$, $C_m \ominus P_n$, $C_n(Q_m)$, $\left[P_n; C_m^{(2)}\right]$, $C_m *_e C_n$ and the graph obtained by duplicating an arbitrary vertex and edge of a cycle C_n admit an even vertex equitable even labeling. We proved that wheel graph W_n and complete graph $K_n, (n > 3)$ are not an even vertex equitable even graph. Also, we proved that $G_1 * G_2$, bistar $B\,(n, n+1)$, caterpillar, arbitrary super subdivision of any path, kC_4-snake, $S(D(Q_n))$, $S(D(T_n))$, $DA(Q_m) \odot nK_1$, $DA(T_m) \odot nK_1$, $S(DA(Q_n))$, $S(DA(T_n))$, jewel graph J_n, jelly fish graph $(JF)_n$, balanced lobster $BL(n, 2, m)$, $\langle L_n \hat{o} K_{1,m} \rangle$, tadpole $T(m, n)$ and $K_{1,n} \bigcup K_{1,n+k}$ if $k \in \{1, 2, 3\}$ admit an even vertex equitable even labeling.

In this paper, we prove that $T\hat{o}Q_n$, $T\tilde{o}Q_n$ and H-graph admit an even vertex equitable even labeling. We use the following definitions in the subsequent sections.

Definition 1.1. *Let G be a graph with p vertices and q edges and $A = \{0, 2, 4, \cdots, q+1\}$ if q is odd or $A = \{0, 2, 4, \cdots, q\}$ if q is even. A graph G is said to be an even vertex equitable even labeling if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2, 4, \cdots, 2q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits even vertex equitable even labeling is called an even vertex equitable even graph.*

Definition 1.2. [3] *Let T be a tree and u_0 and v_0 be two adjacent vertices in T. Let there be two pendant vertices u and v in T such that the length of $u_0 - u$ path is equal to the length of $v_0 - v$ path. If the edge u_0v_0 is deleted from T and u, v are joined by an edge uv, then such a transformation of T is called an elementary parallel transformation (or an ept) and the edge u_0v_0 is called transformable edge.*

If by the sequence of ept's, T can be reduced to a path, then T is called a T_p-tree (transformed tree) and such a sequence regarded as a composition of mappings (ept's) denoted by P, is called a parallel transformation of T. The path, the image of T under P is denoted as $P(T)$.

A T_P-tree and a sequence of two ept's reducing it to a path are shown in Figure 1.

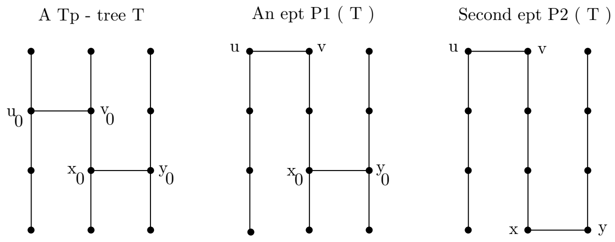

FIGURE 1

Definition 1.3. *Let G_1 be a graph with p vertices and G_2 be any graph. A graph $G_1 \hat{o} G_2$ is obtained from G_1 and p copies of G_2 by identifying one vertex of i^{th} copy of G_2 with i^{th} vertex of G_1.*

Definition 1.4. *Let G_1 be a graph with p vertices and G_2 be any graph. A graph $G_1 \tilde{o} G_2$ is obtained from G_1 and p copies of G_2 by joining one vertex of i^{th} copy of G_2 with i^{th} vertex of G_1 by an edge.*

Definition 1.5. *The H-graph is the graph obtained from two copies of P_n with vertices $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ by joining the vertices $u_{\lfloor \frac{n}{2} \rfloor}$ and $v_{\lceil \frac{n}{2} \rceil}$. It is denoted by H_n.*

2. TREE RELATED GRAPHS

Theorem 2.1. *Let T be a T_p-tree on m vertices. Then the graph $T \hat{o} Q_n$ is an even vertex equitable even graph.*

Proof. Let T be a T_p-tree with m vertices. By the definition of a transformed tree there exists a parallel transformation P of T such that for the path $P(T)$, we have $(i) V(P(T)) = V(T)$ and $(ii) E(P(T)) = (E(T) - E_d) \bigcup E_p$, where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \cdots, P_k)$ of the *epts* P used to arrive at the path $P(T)$. Clearly, E_d and E_p have the same number of edges. Denote the vertices of $P(T)$ successively as $v_1, v_2, \cdots, v_m$ starting from one pendant vertex of $P(T)$ right up to the other. Let $u_1^j, u_2^j, \cdots, u_n^j, u_{n+1}^j (1 \leq j \leq m)$ be the vertices of j^{th} copy of Q_n with $u_{n+1}^j = v_j$. Then $V(T \hat{o} Q_n) = \{u_i^j : 1 \leq i \leq n+1, 1 \leq j \leq m\} \bigcup \{x_i^j, y_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(T \hat{o} Q_n) = E(T) \bigcup E(Q_n)$. We note that $|V(T \hat{o} Q_n)| = m(3n+1)$ and $|E(T \hat{o} Q_n)| = 4mn + m - 1$. Define

$$f : V(T \hat{o} Q_n) \to A = \begin{cases} 0, 2, \cdots, 4mn + m & \text{if } 4mn + m - 1 \text{ is odd} \\ 0, 2, \cdots, 4mn + m - 1 & \text{if } 4mn + m - 1 \text{ is even} \end{cases}$$

as follows:

For $1 \leq i \leq n+1$,

$$f(u_i^j) = \begin{cases} (4n+1)(j-1) + 4(i-1) & \text{if } j \text{ is odd and } 1 \leq j \leq m \\ (4n+1)j - 4(i-1) & \text{if } j \text{ is even and } 1 \leq j \leq m \text{ ;} \end{cases}$$

$$f(v_j) = f(u_n^j);$$

For $1 \leq i \leq n$,

$$f(x_i^j) = \begin{cases} (4n+1)(j-1)+4i & \text{if } j \text{ is odd and } 1 \leq j \leq m \\ (4n+1)j-4i+2 & \text{if } j \text{ is even and } 1 \leq j \leq m \text{ ;} \end{cases}$$
$$f(y_i^j) = \begin{cases} (4n+1)(j-1)+4(i-1)+2 & \text{if } j \text{ is odd and } 1 \leq j \leq m \\ (4n+1)j-4i & \text{if } j \text{ is even and } 1 \leq j \leq m \text{ .} \end{cases}$$

Let v_iv_j be a transformed edge in T, $1 \leq i < j \leq m$ and let P_1 be the *ept* obtained by deleting the edge v_iv_j and adding the edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent *epts*.

Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i+2t+1$. Therefore, i and j are of opposite parity.
The value of the edge v_iv_j is given by

$$\begin{aligned} f^*(v_iv_j) &= f^*(v_iv_{i+2t+1}) \\ &= f(v_i) + f(v_{i+2t+1}) \\ &= (4n+1)(2i+2t). \end{aligned}$$

The value of the edge $v_{i+t}v_{j-t}$ is given by

$$\begin{aligned} f^*(v_{i+t}v_{j-t}) &= f^*(v_{i+t}v_{i+t+1}) \\ &= f(v_{i+t}) + f(v_{i+t+1}) \\ &= (4n+1)(2i+2t). \end{aligned}$$

Therefore, $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$.
The induced edge labels are
$f^*(v_jv_{j+1}) = (4n+1)(2j),\ 1 \leq j \leq m-1$;
For $1 \leq j \leq m$ and $1 \leq i \leq n$,

$$f^*(u_i^jx_i^j) = \begin{cases} (4n+1)2(j-1)+4(2i-1) & \text{if } j \text{ is odd} \\ (4n+1)2j-4(2i-1)+2 & \text{if } j \text{ is even ;} \end{cases}$$
$$f^*(u_i^jy_i^j) = \begin{cases} (4n+1)2(j-1)+8(i-1)+2 & \text{if } j \text{ is odd} \\ (4n+1)2j-4(2i-1) & \text{if } j \text{ is even ;} \end{cases}$$
$$f^*(x_i^ju_{i+1}^j) = \begin{cases} (4n+1)2(j-1)+8i & \text{if } j \text{ is odd} \\ (4n+1)2j-8i+2 & \text{if } j \text{ is even ;} \end{cases}$$
$$f^*(y_i^ju_{i+1}^j) = \begin{cases} (4n+1)2(j-1)+8i-2 & \text{if } j \text{ is odd} \\ (4n+1)2j-8i & \text{if } j \text{ is even .} \end{cases}$$

Thus, it can be verified that the induced edge labels of $T\hat{o}Q_n$ are $2, 4, \cdots, 8mn+2m-2$ and $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$. Hence, $T\hat{o}Q_n$ is an even vertex equitable even graph. □

Example 2.2. An even vertex equitable even labeling of $T\hat{o}Q_2$ where T is a T_p-tree with 8 vertices is shown in Figure 2.

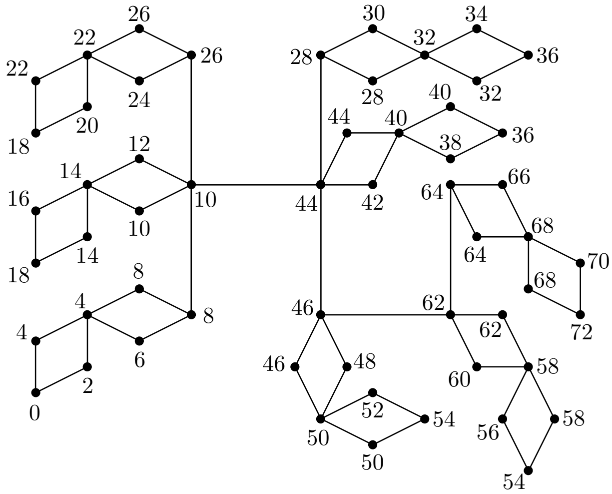

FIGURE 2

Theorem 2.3. *Let T be a T_p-tree on m vertices. Then the graph $T\tilde{o}Q_n$ is an even vertex equitable even graph.*

Proof. Let T be a T_p-tree with m vertices. By the definition of a transformed tree there exists a parallel transformation P of T such that for the path $P(T)$, we have $(i)V(P(T)) = V(T)$ and $(ii)E(P(T)) = (E(T) - E_d) \bigcup E_p$, where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \cdots, P_k)$ of the *epts* P used to arrive at the path $P(T)$. Clearly, E_d and E_p have the same number of edges. Denote the vertices of $P(T)$ successively as $v_1, v_2, \cdots, v_m$ starting from one pendant vertex of $P(T)$ right up to the other. Let $u_1^j, u_2^j, \cdots, u_n^j, u_{n+1}^j (1 \leq j \leq m)$ be the vertices of j^{th} copy of Q_n. Then $V(T\tilde{o}Q_n) = \{v_j, u_i^j : 1 \leq i \leq n+1, 1 \leq j \leq m\} \bigcup \{x_i^j, y_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(T\tilde{o}Q_n) = E(T) \bigcup E(Q_n) \bigcup \{v_j u_{n+1}^j : 1 \leq j \leq m\}$. We note that $|V(T\tilde{o}Q_n)| = m(3n+2)$ and $|E(T\tilde{o}Q_n)| = 4mn + 2m - 1$. Define

$$f : V(T\tilde{o}Q_n) \to A = \begin{cases} 0, 2, \cdots, 4mn + 2m & \text{if } 4mn + 2m - 1 \text{ is odd} \\ 0, 2, \cdots, 4mn + 2m - 1 & \text{if } 4mn + 2m - 1 \text{ is even} \end{cases}$$

as follows:

$$f(v_j) = \begin{cases} (4n+2)j & \text{if } j \text{ is odd and } 1 \leq j \leq m \\ (4n+2)(j-1) & \text{if } j \text{ is even and } 1 \leq j \leq m\ ; \end{cases}$$

For $1 \leq i \leq n+1$,

$$f(u_i^j) = \begin{cases} (4n+2)(j-1) + 4(i-1) & \text{if } j \text{ is odd and } 1 \leq j \leq m \\ (4n+2)j - 4(i-1) & \text{if } j \text{ is even and } 1 \leq j \leq m\ ; \end{cases}$$

For $1 \leq i \leq n$,

$$f(x_i^j) = \begin{cases} (4n+2)(j-1) + 4i & \text{if } j \text{ is odd and } 1 \leq j \leq m \\ (4n+2)j - 4i + 2 & \text{if } j \text{ is even and } 1 \leq j \leq m\ ; \end{cases}$$

$$f(y_i^j) = \begin{cases} (4n+2)(j-1)+4(i-1)+2 & \text{if } j \text{ is odd and } 1 \le j \le m \\ (4n+2)j-4i & \text{if } j \text{ is even and } 1 \le j \le m\ . \end{cases}$$

Let v_iv_j be a transformed edge in T, $1 \le i < j \le m$ and let P_1 be the *ept* obtained by deleting the edge v_iv_j and adding the edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent *epts*. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i+2t+1$. Therefore, i and j are of opposite parity.
The value of the edge v_iv_j is given by

$$\begin{aligned} f^*(v_iv_j) &= f^*(v_iv_{i+2t+1}) \\ &= f(v_i) + f(v_{i+2t+1}) \\ &= (4n+2)(2i+2t). \end{aligned}$$

The value of the edge $v_{i+t}v_{j-t}$ is given by

$$\begin{aligned} f^*(v_{i+t}v_{j-t}) &= f^*(v_{i+t}v_{i+t+1}) \\ &= f(v_{i+t}) + f(v_{i+t+1}) \\ &= (4n+2)(2i+2t). \end{aligned}$$

Therefore, $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$.
The induced edge labels are
$f^*(v_jv_{j+1}) = (4n+1)(2j),\ 1 \le j \le m-1;$
For $1 \le j \le m$ and $1 \le i \le n$,

$$f^*(u_i^jx_i^j) = \begin{cases} (4n+2)2(j-1)+4(2i-1) & \text{if } j \text{ is odd} \\ (4n+2)2j-4(2i-1)+2 & \text{if } j \text{ is even ;} \end{cases}$$

$$f^*(u_i^jy_i^j) = \begin{cases} (4n+2)2(j-1)+8(i-1)+2 & \text{if } j \text{ is odd} \\ (4n+2)2j-4(2i-1) & \text{if } j \text{ is even ;} \end{cases}$$

$$f^*(x_i^ju_{i+1}^j) = \begin{cases} (4n+2)2(j-1)+8i & \text{if } j \text{ is odd} \\ (4n+2)2j-8i+2 & \text{if } j \text{ is even ;} \end{cases}$$

$$f^*(y_i^ju_{i+1}^j) = \begin{cases} (4n+2)2(j-1)+8i-2 & \text{if } j \text{ is odd} \\ (4n+2)2j-8i & \text{if } j \text{ is even ;} \end{cases}$$

$$f^*(v_ju_{n+1}^j) = \begin{cases} (4n+2)(2j-1)+4n & \text{if } j \text{ is odd} \\ (4n+2)(2j-1)-4n & \text{if } j \text{ is even .} \end{cases}$$

Thus, it can be verified that the induced edge labels of $T\tilde{o}Q_n$ are $2, 4, \cdots, 8mn+4m-2$ and $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$. Hence, $T\tilde{o}Q_n$ is an even vertex equitable even graph. □

Example 2.4. An even vertex equitable even labeling of $T\tilde{o}Q_2$ where T is a T_p-tree with 8 vertices is shown in Figure 3.

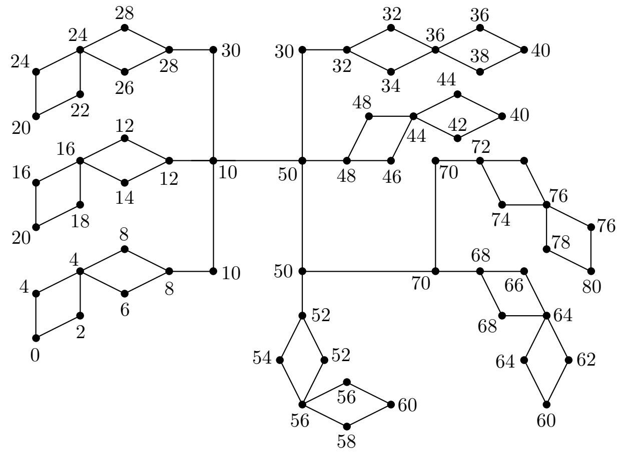

FIGURE 3

Theorem 2.5. *The H-graph is an even vertex equitable even graph.*

Proof. Let $V(H_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(H_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_i v_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_{\lfloor \frac{n}{2} \rfloor} v_{\lceil \frac{n}{2} \rceil}\}$. Then H_n is of order $2n$ and size $2n-1$. Define $f : V(H_n) \to A = \{0, 2, 4, \cdots, 2n+2\}$ as follows:

$$f(u_i) = \begin{cases} 2i-2, & \text{if } i \text{ is odd and } 1 \leq i \leq n \\ 2i, & \text{if } i \text{ is even and } 1 \leq i \leq n \,; \end{cases}$$

$$f(v_i) = \begin{cases} 2n-(i-1), & \text{if } i \text{ is odd and } 1 \leq i \leq n \\ 2n-i, & \text{if } i \text{ is even and } 1 \leq i \leq n \,. \end{cases}$$

Then the induced edge labels are

$f(u_i u_{i+1}) = 2i, \ 1 \leq i \leq n-1;$

$f(v_i v_{i+1}) = 2n - 2(i-1), \ 1 \leq i \leq n-1;$

$f(u_{\lfloor \frac{n}{2} \rfloor} v_{\lceil \frac{n}{2} \rceil}) = 2n.$

Thus, it can be verified that the induced edge labels of H_n are $2, 4, \cdots, 4n-2$ and $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$. Hence, H_n is an even vertex equitable even graph. □

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